

# Comparison of Australian Curriculum Mathematics Specialist course with current Western Australian Mathematics courses

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## Abstract

This paper was prepared for a presentation entitled “Mathematics Specialist for First Timers (and We’re All First Timers)” at the 2014 State Conference of the Mathematics Association of Western Australia. In it we draw links between the current Western Australian Mathematics and Mathematics Specialist courses and the Australian Curriculum Mathematics Specialist course to be introduced in Western Australia from next year.

## 1 Overview

The curriculum document gives the following overview of the AC Mathematics Specialist course (with the added emphasis being ours):

Mathematics Specialist is an ATAR course which provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop **rigorous mathematical arguments and proofs**, and to use **mathematical models** more extensively. The Mathematics Specialist ATAR course contains topics in **functions and calculus** that build on and deepen the ideas presented in the Mathematics Methods ATAR course, as well as demonstrate their **application** in many areas. This course also extends understanding and knowledge of **statistics** and introduces the topics of **vectors, complex numbers** and **matrices**. The Mathematics Specialist ATAR course is the only ATAR mathematics course that should not be taken as a stand-alone course.

From this first overview we can see that there is substantial overlap between the new course and the existing Mathematics Specialist course. The inclusion of statistics, however, tells us that the overlap is not complete and anyone taking the course will need to learn to present a different combination of subjects than any that has been taught in the past.

## 2 Rationale

Before we look at the specifics of the course content, let's look at the first few paragraphs of the Rationale from the curriculum document:

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

This first paragraph is a generic statement about the utility of mathematics and statistics for modelling and for structuring thought. Nothing surprising or challenging here.

Because both mathematics and statistics are widely applicable as models of the world around us, there is ample opportunity for problem-solving throughout the Mathematics Specialist ATAR course. There is also a sound logical basis to this course, and in mastering the course, students will develop logical reasoning skills to a high level.

The emphasis on problem-solving and logical reasoning is again unsurprising. It's what we expect mathematics at the top level to be about and is something that any teacher with a mathematics major should be comfortable with.

The Mathematics Specialist ATAR course provides opportunities, beyond those presented in the Mathematics Methods ATAR course, to develop rigorous mathematical arguments and proofs, and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Students of the Mathematics Specialist ATAR course will be encouraged to appreciate the true nature of mathematics, its beauty and its functionality.

Note that the emphasis is not strictly utilitarian. Some of the skills and methods at this level can be challenging but it's important in working through these difficult topics that we not lose sight of this intention of leading students to appreciate the beauty of mathematics. It's this emphasis that makes teaching Specialist exciting: it's the course that really lets us focus on the beauty that led us to become teachers of mathematics.

## 3 The Topics

### 3.1 Unit 1

#### 3.1.1 Topic 1.1: Combinatorics (11 hours)

Some of this topic overlaps the current 3CMAT content, some is not found in any current course.

**Permutations (ordered arrangements)** 1.1.1 solve problems involving permutations; 1.1.2 use the multiplication and addition principle; 1.1.3 use factorial notation and  ${}^nP_r$ ; 1.1.4 solve problems involving permutations involving restrictions with or without repeated objects

The ‘permutation’ is not found in the current courses, but many of the ideas are included. For example, the multiplication and addition principle is mentioned in 3AMAT item 3.1.4.

**The inclusion-exclusion principle for the union of two sets and three sets** 1.1.5 determine and use the formulas for finding the number of elements in the union of two and the union of three sets

3AMAT covers the appropriate set notation, and 3CMAT covers the union of two sets (couched in terms of probability). Union of three sets is new.

**The pigeon-hole principle** 1.1.6 solve problems and prove results using the pigeon-hole principle

The pigeon-hole principle is not specifically included in any current course.

**Combinations (unordered selections)** 1.1.7 solve problems involving combinations; 1.1.8 use the notation  $\binom{n}{r}$  or  ${}^nC_r$ ; 1.1.9 derive and use associated simple identities associated with Pascal’s triangle

Most of the Combinations items are included in the 3CMAT content, but identities associated with Pascal’s triangle are not in any current course.

**Resources for topic 1.1:** Existing resources for the 3CMAT course will be of some use, particularly in dealing with combinations. Some older resources for Applicable Mathematics may also be useful.

#### 3.1.2 Topic 1.2: Vectors in the plane (22 hours)

All the content dealing with vectors in the plane overlaps existing 3AMAS and 3BMAS content. Some of the emphasis is different, however. For example, 3AMAS refers to projection of vectors merely as a way of introducing the scalar product while the AC content lists it as 1.2.13. (Note that the new course does not have the same depth in year 11 as 3AMAS/3BMAS. Some of the 3BMAS content relating to vectors—the vector equation of a line, for example—is deferred to Unit 3.)

**Representing vectors in the plane by directed line segments** 1.2.1 examine examples of vectors, including displacement and velocity; 1.2.2 define and use the magnitude and direction of a vector; 1.2.3 represent a scalar multiple of a vector; 1.2.4 use the triangle and parallelogram rules to find the sum and difference of two vectors

**Algebra of vectors in the plane** 1.2.5 use ordered pair notation and column vector notation to represent a vector; 1.2.6 define unit vectors and the perpendicular unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ ; 1.2.7 express a vector in component form using the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ ; 1.2.8 examine and use addition and subtraction of vectors in component form; 1.2.9 define and use multiplication of a vector by a scalar in component form; 1.2.10 define and use scalar (dot) product; 1.2.11 apply the scalar product to vectors expressed in component form; 1.2.12 examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular; 1.2.13 define and use projection of vectors; 1.2.14 solve problems involving displacement, force and velocity involving the above concepts

**Resources for topic 1.2:** Existing resources for the 3AMAS and 3BMAS course will cover almost all the content for topic 1.2, but may need to be supplemented here and there to reflect the different emphasis.

### 3.1.3 Topic 1.3: Geometry (22 hours)

**The nature of proof** 1.3.1 use implication, converse, equivalence, negation, inverse, contrapositive; 1.3.2 use proof by contradiction; 1.3.3 use the symbols for implication ( $\implies$ ), equivalence ( $\iff$ ); 1.3.4 use the quantifiers ‘for all’  $\forall$  and ‘there exists’  $\exists$ ; 1.3.5 use examples and counter-examples

The AC Specialist course takes a much more formal approach to mathematical proof than is found in any of the current courses. It explicitly addresses the language and notation of mathematical proof. This is not mentioned in the current courses (although it seems likely that many teachers will have been including this as part of the way they have gone about dealing with the Mathematical Reasoning content in the current Specialist course).

**Circle properties, including proof and use** 1.3.6 an angle in a semicircle is a right angle; 1.3.7 the size of the angle at the centre subtended by an arc of a circle is twice the size of the angle at the circumference subtended by the same arc; 1.3.8 angles at the circumference of a circle subtended by the same arc are equal; 1.3.9 the opposite angles of a cyclic quadrilateral are supplementary; 1.3.10 chords of equal length subtend equal angles at the centre, and conversely, chords subtending equal angles at the centre of a circle have the same length; 1.3.11 the angle in the alternate segment theorem; 1.3.12 when two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord; 1.3.13 when a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn to a circle from an external point M, the square of length of the tangent equals the product of the lengths to the circle on the secant ( $AM \times BM = TM^2$ ); 1.3.14 suitable converses of some of the above results; 1.3.15 solve problems determining unknown angles and lengths and prove further results using the results listed above

These circle theorems and their proof are almost unlike anything in any of the current courses, although they are far from new since most (if not all) of them can be found in Euclid’s *Elements*. Some of them could be part of 3DMAT (item 2.3 Reason Geometrically) but with none of the specificity and a much smaller emphasis than is included here.

**Geometric vectors in the plane, including proof and use** 1.3.16 the diagonals of a parallelogram intersect at right angles if, and only if, it is a

rhombus; 1.3.17 the midpoints of the sides of a quadrilateral join to form a parallelogram; 1.3.18 the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides

Vector proofs are included in 3CMAS although without the level of detail specified here. Although the 3CMAS entry is only one point (rather than the three listed here) it would be expected that students of the current course would be able to address all these points and many more. These three points should probably be taken as descriptive rather than prescriptive and the vector proof content of the AC course should be understood to be much the same as the current 3CMAS course.

**Resources for Topic 1.3:** Dr Paul Brown's 2010 book *Proof: Interesting Activities in Conjecture and Mathematical Proof* remains a valuable resource for the underlying ideas of mathematical proof (ISBN 9780980819601; see <http://www.pbperth.com>). Existing resources for 3DMAT and 3CMAS will cover much of the content here, although additional student problems may need to be sourced to fulfil the increased emphasis.

## 3.2 Unit 2

### 3.2.1 Topic 2.1: Trigonometry (16 hours)

#### The basic trigonometric functions

This work extends the AC Mathematical Methods topic 1.2 and is included in the current 3BMAS course. (Much of the current MAS content relating to Trigonometry now appears in Methods, not Specialist.)

2.1.1 determine all solutions of  $f(a(x - b)) = c$  where  $f$  is one of sine, cosine or tangent; 2.1.2 graph functions with rules of the form  $y = f(a(x - b)) + c$  where  $f$  is one of sine, cosine, or tangent

**Compound angles** 2.1.3 prove and apply the angle sum, difference, and double angle identities

(This seems to be a near duplicate of Mathematical Methods item 1.2.14.)

**The reciprocal trigonometric functions, secant, cosecant and cotangent** 2.1.4 define the reciprocal trigonometric functions; sketch their graphs and graph simple transformations of them

These reciprocal functions are not part of the syllabus for any of the current courses (although I think they are usually covered in passing by teachers of the Specialist course).

**Trigonometric identities** 2.1.5 prove and apply the Pythagorean identities; 2.1.6 prove and apply the identities for products of sines and cosines expressed as sums and differences; 2.1.7 convert sums  $a \cos x + b \sin x$  to  $R \cos(x \pm \alpha)$  or  $R \sin(x \pm \alpha)$  and apply these to sketch graphs; solve equations of the form  $a \cos x + b \sin x = c$ ; 2.1.8 prove and apply other trigonometric identities such as  $\cos 3x = 4 \cos^3 x - 3 \cos x$

Proofs relating to trigonometric identities are covered in both 3BMAS and 3CMAS.

**Applications of trigonometric functions to model periodic phenomena** 2.1.9 model periodic motion using sine and cosine functions and understand

the relevance of the period and amplitude of these functions in the model

This kind of modelling with trigonometric functions is part the overall thrust of the current MAS coursework and particular focus on amplitude, period and phase is included in 3BMAS.

**Resources for topic 2.1:** Apart from the inclusion of reciprocal functions, the remainder of this topic will be covered well by current MAS resources, mostly from unit 3BMAS.

### 3.2.2 Topic 2.2: Matrices (19 hours)

**Matrix arithmetic** 2.2.1 apply matrix definition and notation; 2.2.2 define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity, and inverse; 2.2.3 calculate the determinant and inverse of  $2 \times 2$  matrices and solve matrix equations of the form  $AX = B$ , where  $A$  is a  $2 \times 2$  matrix and  $X$  and  $B$  are column vectors

**Transformations in the plane** 2.2.4 examine translations and their representation as column vectors; 2.2.5 define and use basic linear transformations: dilations of the form  $(x, y) \rightarrow (\lambda_1 x, \lambda_2 y)$ , rotations about the origin and reflection in a line that passes through the origin and the representations of these transformations by  $2 \times 2$  matrices; 2.2.6 apply these transformations to points in the plane and geometric objects; 2.2.7 define and use composition of linear transformations and the corresponding matrix products; 2.2.8 define and use inverses of linear transformations and the relationship with the matrix inverse; 2.2.9 examine the relationship between the determinant and the effect of a linear transformation on area; 2.2.10 establish geometric results by matrix multiplications; for example: show that the combined effect of 2 reflections is a rotation

**Systems of linear equations** 2.2.11 interpret the matrix form of a system of linear equations in two variables and use matrix algebra to solve a system of linear equations

This whole topic is covered by the current 3DMAS course.

**Resources for topic 2.2:** This topic is well covered by existing 3DMAS resources.

### 3.2.3 Topic 2.3: Real and complex numbers (20 hours)

**Proofs involving numbers** 2.3.1 prove simple results involving numbers

**Rational and irrational numbers** 2.3.2 express rational numbers as terminating or eventually recurring decimals and vice versa; 2.3.3 prove irrationality by contradiction for numbers such as  $\sqrt{2}$

The proof aspects of the above are covered in the 3CMAS course.

**An introduction to proof by mathematical induction** 2.3.4 develop the nature of inductive proof, including the ‘initial statement’ and inductive step; 2.3.5 prove results for sums, such as  $1 + 4 + 9 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$  for any positive integer  $n$ ; 2.3.6 prove divisibility results, such as  $3^{2n+4} - 3^{2n}$  is divisible by 5 for any positive integer  $n$

Proof by the principle of mathematical induction is covered in the 3DMAS course.

**Complex numbers** 2.3.7 define the imaginary number  $i$  as a root of the equation  $x^2 = -1$ ; 2.3.8 represent complex numbers in the rectangular form;  $a + bi$  where  $a$  and  $b$  are the real and imaginary parts; 2.3.9 determine and use complex conjugates; 2.3.10 perform complex number arithmetic: addition, subtraction, multiplication and division

**The complex plane** 2.3.11 consider complex numbers as points in a plane, with real and imaginary parts, as Cartesian coordinates; 2.3.12 examine addition of complex numbers as vector addition in the complex plane; 2.3.13 develop and use the concept of complex conjugates and their location in the complex plane

**Roots of equations** 2.3.14 use the general solution of real quadratic equations; 2.3.15 determine complex conjugate solutions of real quadratic equations; 2.3.16 determine linear factors of real quadratic polynomials

The complex number content is included in the current 3BMAS, possibly excepting 2.3.16 (which is a simple extension of 2.3.14 and 2.3.15).

**Resources for topic 2.3:** Selected resources from 3BMAS, 3CMAS and 3DMAS can be used to address almost all of topic 2.3, possibly excepting 2.3.16.

### 3.3 Unit 3

#### 3.3.1 Topic 3.1: Complex numbers (18 hours)

**Cartesian forms** 3.1.1 review real and imaginary parts  $\text{Re}(z)$  and  $\text{Im}(z)$  of a complex number  $z$ ; 3.1.2 review Cartesian form; 3.1.3 review complex arithmetic using Cartesian forms

**Complex arithmetic using polar form** 3.1.4 use the modulus  $|z|$  of a complex number  $z$  and the argument  $\text{Arg}(z)$  of a non-zero complex number  $z$  and prove basic identities involving modulus and argument; 3.1.5 convert between Cartesian and polar form; 3.1.6 define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these; 3.1.7 prove and use De Moivre's theorem for integral powers

**The complex plane (The Argand plane)** 3.1.8 examine and use addition of complex numbers as vector addition in the complex plane; 3.1.9 examine and use multiplication as a linear transformation in the complex plane; 3.1.10 identify subsets of the complex plane determined by relations such as  $|z - 3i| \leq 4$ ,  $\frac{\pi}{4} \leq \text{Arg}(z) \leq \frac{3\pi}{4}$  and  $|z - 1| = 2|z - 1|$

**Roots of complex numbers** 3.1.11 determine and examine the  $n^{\text{th}}$  roots of unity and their location on the unit circle; 3.1.12 determine and examine the  $n^{\text{th}}$  roots of complex numbers and their location in the complex plane

The complex number content to this point is all covered in the current 3CMAS and 3DMAS courses.

**Factorisation of polynomials** 3.1.13 prove and apply the factor theorem and the remainder theorem for polynomials; 3.1.14 consider conjugate roots for polynomials with real coefficients; 3.1.15 solve simple polynomial equations

Factorisation of polynomials of degree greater than two is not included in any current courses. Conjugate roots of quadratics are considered in 3BMAS. Solving polynomials of degree greater than two is not included in any current course unless the polynomial is already in factor form.

**Resources for topic 3.1:** Apart from some of the work on factorization of

polynomials, the complex number content is covered well in existing resources for 3CMAS and 3DMAS. Resources for the pre-2009 Calculus course can provide most what is needed for factorization of polynomials.

### 3.3.2 Topic 3.2: Functions and sketching graphs (16 hours)

**Functions** 3.2.1 determine when the composition of two functions is defined; 3.2.2 determine the composition of two functions; 3.2.3 determine if a function is one-to-one; 3.2.4 find the inverse function of a one-to-one function; 3.2.5 examine the reflection property of the graphs of a function and its inverse

The work on composite and inverse functions is included in the current 3CMAT and 3AMAS courses.

**Sketching graphs** 3.2.6 use and apply  $|x|$  for the absolute value of the real number  $x$  and the graph of  $y = |x|$ ; 3.2.7 examine the relationship between the graph of  $y = f(x)$  and the graphs of  $y = \frac{1}{f(x)}$ ,  $y = |f(x)|$  and  $y = f(|x|)$ ; 3.2.8 sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree

The current 3AMAS course has similar content relating to absolute values and covers  $y = |f(x)|$ , but graphing  $y = \frac{1}{f(x)}$  and  $y = f(|x|)$  and graphs of rational functions are not included in any of the current courses.

**Resources for topic 3.2:** Much of this topic is covered in the existing 3AMAS course and the 3CMAT course also covers some of the composition of functions work (but with a more restricted range of functions).

### 3.3.3 Topic 3.3: Vectors in three dimensions (21 hours)

**The algebra of vectors in three dimensions** 3.3.1 review the concepts of vectors from Unit 1 and extend to three dimensions, including introducing the unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ ; 3.3.2 prove geometric results in the plane and construct simple proofs in 3 dimensions

This section is all included in the content of the current 3CMAS course.

**Vector and Cartesian equations** 3.3.3 introduce Cartesian coordinates for three dimensional space, including plotting points and equations of spheres; 3.3.4 use vector equations of curves in two or three dimensions involving a parameter and determine a ‘corresponding’ Cartesian equation in the two-dimensional case; 3.3.5 determine a vector equation of a straight line and straight line segment, given the position of two points or equivalent information, in both two and three dimensions; 3.3.6 examine the position of two particles, each described as a vector function of time, and determine if their paths cross or if the particles meet; 3.3.7 use the cross product to determine a vector normal to a given plane; 3.3.8 determine vector and Cartesian equations of a plane



Cartesian coordinates for 3D space are included in the current 3CMAS course. Vector equation of a line and a circle in the plane are included in the current 3BMAS course. Converting between vector, parametric and Cartesian equations is included in the 3CMAS course for equations of lines only, so item 3.3.4 is a little more general than anything in current courses. Examining the paths of two particles for intersection of paths or collision/interception is included in 3BMAS (for motion in the plane) and 3CMAS (for motion in space). (The current course covers rather more in this area than is included here.) Vector and Cartesian equations of a plane is covered in 3CMAS.

None of the current courses cover vector cross product.

**Systems of linear equations** 3.3.9 recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations; 3.3.10 examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables

Solving systems of simultaneous equations in two variables algebraically and graphically is included in 2BMAT, 2DMAT and 3BMAT. Solving systems of three variables by elimination is included in 3DMAT. Geometric interpretation and examination of the three cases for solutions of systems of equations are not explicitly included in the current course.

**Vector calculus** 3.3.11 consider position vectors as a function of time; 3.3.12 derive the Cartesian equation of a path given as a vector equation in two dimensions, including ellipses and hyperbolas; 3.3.13 differentiate and integrate a vector function with respect to time; 3.3.14 determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration; 3.3.15 apply vector calculus to motion in a plane, including projectile and circular motion

There is no Vector calculus in any of the current courses.

**Resources for topic 3.3:** Existing resources for 3BMAS and 3CMAS will cover most of the vector content with the notable exception of vector cross products. Existing resources for 3BMAT and 3DMAT will cover most of the techniques required for systems of linear equations, although they may need to be expanded to properly meet the requirements of 3.3.10.

None of the resources for existing courses will address the Vector Calculus requirement. Older resources from the pre-2009 Calculus course can be used to address these.

## 3.4 Unit 4

### 3.4.1 Topic 4.1: Integration and applications of integration (20 hours)

**Integration techniques** 4.1.1 integrate using the trigonometric identities  $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ ,  $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$  and  $1 + \tan^2 x = \sec^2 x$ ; 4.1.2 use substitution  $u = g(x)$  to integrate expressions of the form  $f(g(x))g'(x)$ ; 4.1.3 establish and use the formula  $\int \frac{1}{x} dx = \ln |x| + c$  for  $x \neq 0$ ; 4.1.4 use partial fractions where necessary for integration in simple cases

Many of these integration techniques are covered by existing 3CMAS resources. Partial fraction decomposition is not part of any current course.

**Applications of integral calculus** 4.1.5 calculate areas between curves determined by functions; 4.1.6 determine volumes of solids of revolution about either axis; 4.1.7 use technology with numerical integration

Area between curves is covered in 3CMAT (for polynomial functions) and 3CMAS (for other functions) and solids of revolution are covered in 3DMAT. Numerical integration using technology is not explicit in any current course, although it may be implied in 3BMAS and 3CMAS.

**Resources for topic 4.1:** A fair amount of this topic is covered by existing resources for 3CMAS and 3DMAS, but none of the resources for existing or past courses cover partial fractions.

### 3.4.2 Topic 4.2: Rates of change and differential equations (20 hours)

**Applications of differentiation** 4.2.1 use implicit differentiation to determine the gradient of curves whose equations are given in implicit form; 4.2.2 examine related rates as instances of the chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ ; 4.2.3 apply the incremental formula  $\delta y \approx \frac{dy}{dx} \delta x$  to differential equations; 4.2.4 solve simple first order differential equations of the form  $\frac{dy}{dx} = f(x)$ ; differential equations of the form  $\frac{dy}{dx} = g(y)$ ; and, in general, differential equations of the form  $\frac{dy}{dx} = f(x)g(y)$ , using separation of variables; 4.2.5 examine slope (direction or gradient) fields of a first order differential equation; 4.2.6 formulate differential equations, including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved

Implicit differentiation is covered in 3CMAS; related rates, incremental formulas and solving differential equations (variables separable) are included in 3DMAS. The rest, including slope fields and formulation of differential equations (including the logistic equation) is outside the scope of any current course.

**Modelling motion** 4.2.7 consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions  $\frac{dv}{dt}$ ,  $v \frac{dv}{dx}$  and  $\frac{d}{dx} \left( \frac{1}{2} v^2 \right)$  for acceleration

Simple Harmonic Motion and other examples of motion in a line are covered in the current 3DMAS course but use of different expressions for acceleration is not explicit in any current course.

**Resources for topic 4.2:** Much of this topic is covered by existing resources for 3DMAS, but the topic takes differential equations further than current courses and is more explicit about some of the content for motion in a line.

### 3.4.3 Topic 4.3: Statistical inference (15 hours)

**Sample means** 4.3.1 examine the concept of the sample mean  $\bar{X}$  as a random variable whose value varies between samples where  $X$  is a random variable with mean  $\mu$  and the standard deviation  $\sigma$ ; 4.3.2 simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties

of the distribution of  $\bar{X}$  across samples of a fixed size  $n$ , including its mean  $\mu$  its standard deviation  $\frac{\sigma}{\sqrt{n}}$  (where  $\mu$  and  $\sigma$  are the mean and standard deviation of  $X$ ), and its approximate normality if  $n$  is large; 4.3.3 simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of  $\frac{\bar{X} - \mu}{s/\sqrt{n}}$  for large samples ( $n \geq 30$ ), where  $s$  is the sample standard deviation

**Confidence intervals for means** 4.3.4 examine the concept of an interval estimate for a parameter associated with a random variable; 4.3.5 examine the approximate confidence interval  $\left(\bar{X} - \frac{zs}{\sqrt{n}}, \bar{X} + \frac{zs}{\sqrt{n}}\right)$  as an interval estimate for the population mean  $\mu$ , where  $z$  is the appropriate quantile for the standard normal distribution; 4.3.6 use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain  $\mu$ ; 4.3.7 use  $\bar{x}$  and  $s$  to estimate  $\mu$  and *sigma* to obtain approximate intervals covering desired proportions of values of a normal random variable, and compare with an approximate confidence interval for  $\mu$

Although worded differently the content on sample means and confidence intervals for means is similar to that included in the current 3DMAT course.

**Resources for topic 4.3:** This topic is covered by existing resources for 3DMAT.

## 4 Summary

The table below attempts to summarize where topics in the WA version of the Australian Curriculum for Mathematics Specialist overlap current resources for MAT or MAS courses. We have also tried to indicate where there is content that is not covered in the current courses but is covered in the older Calculus course, and which topics include content that is not covered in any WA course this century.

| Topic                | MAT   |       | MAS   |       | Calculus | None |
|----------------------|-------|-------|-------|-------|----------|------|
|                      | 3A/3B | 3C/3D | 3A/3B | 3C/3D |          |      |
| 1.1 Combinatorics    | Some  | Some  |       |       |          | Some |
| 1.2 Vectors (2D)     |       |       | All   |       |          |      |
| 1.3 Geometry         |       | Some  |       | Some  |          | Some |
| 2.1 Trigonometry     |       |       | Most  | Some  |          |      |
| 2.2 Matrices         |       |       |       | All   |          |      |
| 2.3 Real & Complex   | Some  | Most  |       |       |          |      |
| 3.1 Complex Nos.     |       |       |       | Most  | Some     | Some |
| 3.2 Functions        |       | Some  | Some  |       |          |      |
| 3.3 Vectors (3D)     |       | Some  | Some  | Most  | Some     | Some |
| 4.1 Integration      |       |       |       | Most  |          | Some |
| 4.2 R.o.C. and D.E.s |       |       |       | Most  |          | Some |
| 4.3 Stat. Inference  |       | All   |       |       |          |      |